SUBJECT: MATHEMATICS

CLASS XI

1. How many terms of the AP -6,11/2,-5 are needed to give the sum -25?
2. In an AP the 1st term is 2 and the sum of the first 5 terms is one fourth of the next 5 terms.Show that 20th term is -112.
3. If the sum of n terms of 2 AP are in the ratio 5n+4 : 9n+6.Find the ratio of their 18th terms
4. In an AP if pth term is 1/q and qth term is 1/p. Prove that sum of 1st pq terms is ½*(pq+1) , where p≠0.
5. The sum of n terms of an AP is $pn+qn^2$ where p & q are constants. Find the common difference.

CLASS :XII

Solved Model Question Paper–1

Time allowed: 3 hours **Maximum marks: 100**

▪ ***General Instructions:***

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three Sections **A, B** and **C**. Section A comprises of **6** questions of **one** mark each; Section B comprises of **13** questions of **four** marks each; and Section C comprises of **7** questions of **six** marks each.*
3. ***All** questions in Section A are to be answered in **one** word, **one** sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in **4** questions of **four** marks each and **2** questions of **six** marks each. You have to attempt only **one** of the alternatives in all such questions.*
5. *Use of calculator is **not** permitted. You may ask for logarithmic tables, if required.*

SECTION–A

Question numbers 1 to 6 carry 1 mark each.

1. If $A=\{3,4,7,9\}$, $B=\{6,7,8,9,12\}$ and R is the relation "is a factor of" from A to B find R.
2. Evaluate $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$
3. Find the value of x and y, if $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix}=\begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$
4. If A is a square matrix of order 3 such that $|\text{adj } A| = 225$, find $|A'|$.
5. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}\times\vec{b}|=6$.
6. Write the direction cosines of a vector parallel to the line $\frac{4-x}{2}=\frac{y+3}{3}=\frac{z+2}{6}$.

SECTION–B

Question numbers 7 to 19 carry 4 marks each.

7. Prove that the function $f: N \to N$, defined by $f(x)=x^2+x+1$ is one-one but not onto.
8. Show that $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\sqrt{\frac{x^2+1}{x^2+2}}$

OR

Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$.

9. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)$$

10. Show that $f(x)=|x-3|\ \forall x\in R$, is continuous but not differentiable at $x=3$.

OR

Find the value of a and b such that the function defined by

$$f(x)=\begin{cases} 5 & \text{if } x\le 2 \\ ax+b & \text{if } 2<x<10 \\ 21 & \text{if } x\ge 10 \end{cases}$$ is a continuous function.

11. If $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}$.

12. If $x^y+y^x=a^b$, find $\frac{dy}{dx}$.

13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

14. Evaluate $\int\frac{x^3+x+1}{x^2-1}dx$

OR

Evaluate $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

15. Evaluate $\int\frac{1+\cot x}{x+\log(\sin x)}dx$

16. Evaluate $\int_0^{\frac{\pi}{2}}\frac{x\sin x.\cos x}{\sin^4 x+\cos^4 x}dx$.

17. Find the value of λ for which the four points with position vectors $-\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\lambda\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

18. Find the value of ω so that the lines

$\frac{1-x}{3}=\frac{7y-14}{2\omega}=\frac{z-3}{2}$ and $\frac{7-7x}{3\omega}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

OR

Find the shortest distance between the following lines

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

19. *There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Write the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution. Explain the importance of non-violence in patriotism.*

SECTION–C

Question numbers 20 to 26 carry 6 marks each.

20. Two trusts A and B receive ₹ 70,000 and ₹ 55,000 respectively from Central Government to award prize to persons of a district in three fields agriculture, education and social service. Trust A awarded 10, 5 and 15 persons in the field of agriculture, education and social service respectively while trust B awarded 15, 10 and 5 persons respectively. If all three prizes together amount to ₹ 6,000, then find the amount of each prizes by matrix method.

21. Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

OR

Prove that the surface area of a solid cuboid of square base and given volume is minimum when it is cube.

22. Find the area of the region which is enclosed between the two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.

23. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

OR

Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} + 5\hat{k})$. Also, show that the plane contains the line

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k}).$$

24. Find the particular solution of the differential equation.

$$(x - \sin y)dy + (\tan y)dx = 0, \text{ given that } y = 0 \text{ when } x = 0.$$

25. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 560.00 to invest and has space for at most 20 items. An electronic sewing machine costs him ₹ 360.00 and a manually operated sewing machine ₹ 240.00. He can sell an electronic sewing machine at a profit of ₹ 22.00 and a manually operated sewing machine at a profit of ₹ 18.00. Assuming that he can sell all the items that he can buy. How should he invest his money in order to maximise his profit? Make it as a linear programming problem and solve it graphically.

26. An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motor bike driver are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

Solved Model Question Paper–2

Time allowed: 3 hours **Maximum marks: 100**

General Instructions: *As per given in Model Question Paper (Solved)–1.*

SECTION–A

Question numbers 1 to 6 carry 1 mark each.

1. Show that the function given by $f(x) = 3x + 7$ is strictly increasing on R.
2. Evaluate: $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
3. Find the value of x and y, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.
4. For what value of a, $\begin{vmatrix} 2a & -1 \\ -8 & 3 \end{vmatrix}$ is a singular matrix?
5. If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.
6. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$.

SECTION–B

Question numbers 7 to 19 carry 4 marks each.

7. Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.
8. The function $f(x)$ is defined as $f(x) = \begin{cases} x^2 + ax + b & 0 \le x < 2 \\ 3x + 2 & 2 \le x \le 4 \\ 2ax + 5b & 4 < x \le 8 \end{cases}$

   If $f(x)$ is continuous on [0, 8], find the value of a and b.

   OR

   Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$ with respect to $\cos^{-1} x^2$.

9. If $\log (x^2 + y^2) = 2 \tan^{-1} \frac{y}{x}$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

10. If $x = a(\cos t + t \sin t)$ and $y = b(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

11. Evaluate $\int \frac{\sin x + \cos x}{\sqrt{\sin x . \cos x}} dx$.

OR

Evaluate $\int e^x \left(\frac{x^2+1}{(x+1)^2} \right) dx$.

12. Evaluate $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$.

13. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d} . \vec{c} = 21$.

14. Find the distance between the point P (6, 5, 9) and the plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6).

OR

Find the equation of the perpendicular drawn from the point $P(2, 4, -1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also write down the co-ordinates of the foot of the perpendicular from P to the line.

15. ***A director of selection committee is biased so that he selects his relatives for a job 2 times as likely as others. If there are 2 posts for a job, find the probability distribution for selection of his relatives.***

Is the presence of such type of people in selection committee reasonable?

Which type of values will be demolished here?

16. Evaluate $\int \frac{dx}{\cos(x-a) . \cos(x-b)}$.

17. Using properties of determinant prove that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)\,(4-x)^2.$$

OR

Using properties of determinant prove that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

18. Using differential, find the approximate value of $(.0037)^{1/2}$.

19. Show that the relation R defined by $(a, b)\, R\, (c, d) \Rightarrow ad = bc$ on the set $N \times N$ is an equivalence relation.

SECTION–C

Question numbers 20 to 26 carry 6 marks each.

20. An open box with square base is to made out of a given iron sheet of area 27 m^2. Show that the maximum volume of the box is 13.5 m^3.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere.

21. Two cricket team honoured their players for three values, excellent batting, to the point bowling and unparalleled fielding by giving ₹ x, ₹ y and ₹ z per player respectively. The first team paid respectively 2, 2 and 1 players for the above values with a total prize money of ₹ 11 lakhs, while the second team paid respectively 1, 2 and 2 players for these values with a total prize money of ₹ 9 lakhs. If the total award money of one person each for these values amount to ₹ 6 lakhs, then express the above situation as a matrix equation and find the award money per person for each values.

22. Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

OR

Find the area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

23. In a test, an examinee either knows the answer or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{6}$ and the probability that he copies the answer is $\frac{1}{9}$. The probability that his answer is correct, given that he copied, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

24. Find the co-ordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane $2x - y + z + 3 = 0$.

Find also, the image of the point in the plane.

25. Find the particular solution of the differential equation

$(xdy - ydx)\, y \sin \frac{y}{x} = (ydx - xdy)\, x \cos \frac{y}{x}$, given that $y = \pi$ when $x = 3$.

26. A group of farmers has 50 hectare of land to grow two variety of rice X and Y. The profit from variety X and Y per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds a herbicide has to be used for variety X and Y at rate of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to maintain the quality of rice and protection of environment. How much land should be allocated to each variety of rice so as to maximise the total profit of the farmers?